

COLUMN ADDITION (INTEGERS AND DECIMALS)

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
Students should be able to use place value counters to represent the two numbers and understand the idea of carrying.	Students should draw the manipulatives as a diagram in their books, clearly showing where the values are being carried.	Students should use the ideas in the Physical and Pictorial stages and partition the numbers.	Students should use the traditional column addition method, carrying were appropriate
Example(s)	Example(s) - These should be seen in books		

Calculate $243 + 874$

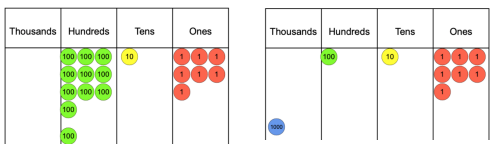
Students should represent the numbers in a place value table using place value counters.



Students should then physically move the counters together starting from the leftmost column.



Students should then understand that 10 1's make 10, 10 10's make 100, etc. and that they can replace these 10 counters with 1 of the next highest counters.

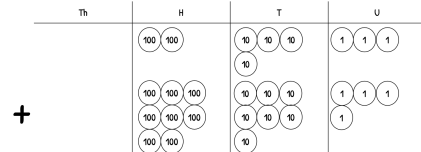


(Notice the added counter is placed lower than the existing ones, this will aid with the idea of carrying later)

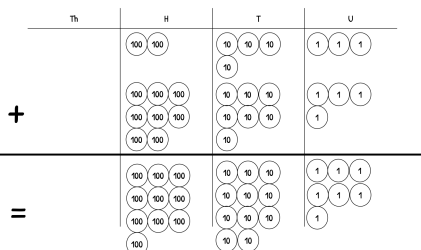
Students can then just count the leftover counters.

$$243 + 874 = 1117$$

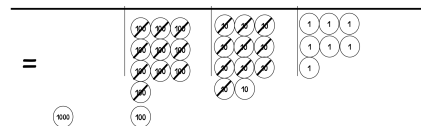
Students should draw the place value table and counters in their books.



Students should then total up each place value column and draw the correct number of each counter.



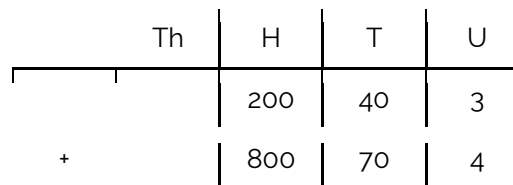
Where there are more than 10 of a place value counter, students should cross out those which would total the next value up and draw an additional counter in the appropriate column, this should be below those counters that are already there.



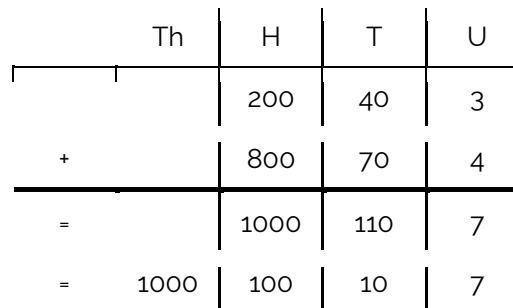
Students can then just count the non-crossed out circles.

$$243 + 874 = 1117$$

Students should partition the numbers and represent these values in a place value table



Students should the sum each column, carrying where appropriate.



Students should then sum the final row.

$$243 + 874 = 1117$$

Students should be able to use a formal column method of addition

$$\begin{array}{r}
 243 \\
 + 874 \\
 \hline
 1117 \\
 1 \quad 1
 \end{array}$$

FACTORS

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
Students should use a manipulative (likely double sided counters or multi-link cubes) and maneuver them to understand factors and multiples	Students use the ideas formed in the physical stage to use the squares in exercise books to draw rectangles	Students should use a diagram to find the factors of the required number	Students should be able to list the factors immediately
Example(s)	Example(s) - These should be seen in books		

Find all of the factors of 12

Students should represent the number using the manipulative



Students should then understand that amount of counters along and the amount of counters up are factors of the value.

Students should then work **systematically**, adding one to the height and attempting to form a rectangle.

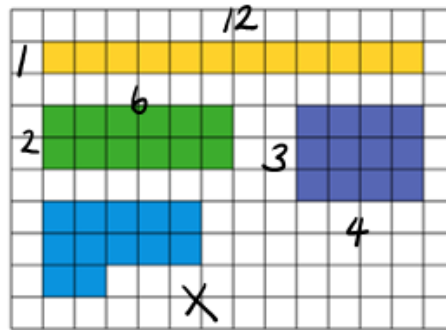
Testing if 1 is a factor: [From this, students should form the understanding that 1 is a factor of every number]			Testing 12: 1 and 12 are factors of 12
Testing if 2 is a factor: Since this forms a perfect rectangle 2 and 6 are factors of 12	Testing if 3 is a factor: Since this forms a perfect rectangle 3 and 4 are factors of 12	Testing if 4 is a factor: Since this forms a perfect rectangle 4 and 3 are factors of 12	
Testing if 4 is a factor: Since this forms a perfect rectangle 4 and 3 are factors of 12	Testing if 5 is a factor: Since this DOES NOT form a perfect rectangle 5 is NOT a factor of 12.	Testing if 6 is a factor: 6 and 2 are factors of 12	

Students should identify that the factors come in pairs and that they only need to be listed once.

Factors of 12 are 1, 2, 3, 4, 6, 12

Students should use the squares in their books to sketch the value in rectangles.

From the Physical stage, students should understand that the factors come in pairs and therefore only need to be drawn once.

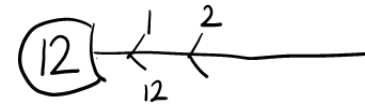


Students should clearly explain why certain values are NOT factors.

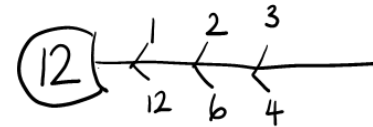
E.g. the light blue diagram above does not form a perfect rectangle, therefore 5 is NOT a factor of 12.

Factors of 12 are 1, 2, 3, 4, 6, 12

Students should work systematically, starting at 1 and increasing by one each time and determining if it is a factor and the corresponding value.



Students should understand that as soon as they reach a number that is already in the diagram they have found all of the factors.



Students MUST then list the factors.

Factors of 12 are 1, 2, 3, 4, 6, 12

Students will not need any form of diagram at this stage.

Students should understand the meaning of factors and will likely be able to find factors in their head.

Factors of 12 are 1, 2, 3, 4, 6, 12

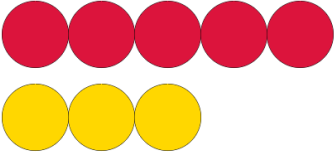
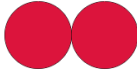


DIRECTED NUMBER

GENERAL TEACHING & LEARNING POINTS

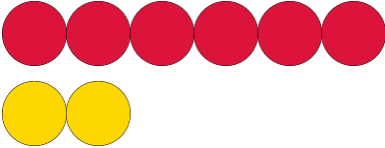

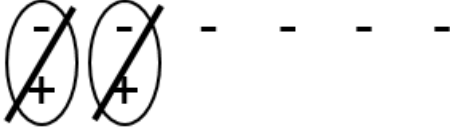
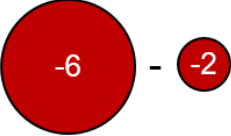

The following apply to all four operations with negative numbers:

- Teachers should not use the term "minus", instead use the words "negative" and "subtract"
- "Two negatives make a positive" should NEVER be used – this is not mathematically accurate
- The idea of zero-pairs should be emphasised throughout as this will also be used in an algebra context
- The idea that subtraction is the **additive inverse** should be shown, e.g. subtracting 3 is the same as adding -3
- Students should be confident with using double-sided counters to represent numbers prior to attempting four operations

Addition

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
Students should be able to use double-sided counters to represent the two numbers and the idea of zero pairs.	Students should draw the manipulatives as a diagram in their books, clearly showing the zero-pairs.	Students should use the ideas in the Physical and Pictorial stages to be able to determine, through using 'scaled' double-sided counter diagrams to determine if the answer is positive or negative.	Students should be able to state an answer without the use of a diagram.
Example(s)	Example(s) - These should be seen in books		
<p>Calculate $-5 + 3$ "Negative 5 add 3"</p>			
<p>Students should use double-sided counters to represent the -5 using five -1 counters and then physically add three +1 counters.</p> <div style="text-align: center;">  </div> <p>Students should then identify the zero-pairs in their representation and remove them.</p> <div style="text-align: center;">  </div> <p>Students should then be able to identify their answer from their remaining counters.</p> <p style="text-align: center;">$-5 + 3 = -2$</p>	<p>Students should represent the calculation as in the Physical Stage, however, for short it may be beneficial for students to use - and + signs instead of drawing full circles.</p> <div style="text-align: center;"> $\begin{array}{cccccc} - & - & - & - & - & \\ + & + & + & & & \end{array}$ </div> <p>Students should then identify the zero-pairs in their representation and circle them. They could show they have been removed by crossing through them.</p> <div style="text-align: center;">  </div> <p>Students should then be able to identify their answer from their remaining symbols.</p> <p style="text-align: center;">$-5 + 3 = -2$</p>	<p>Students should represent this by drawing a larger negative circle plus a smaller positive circle. https://mathsbot.com/manipulatives/directedCounters may be useful for this representation.</p> <p>For weaker students a number line representation in addition may also be beneficial.</p> <div style="text-align: center;">  </div> <p>Students should be able to determine from the calculation that there will be more negative counters than positive ones in this calculation, therefore the answer must be negative.</p> <p style="text-align: center;">$-5 + 3 = -2$</p>	<p>Students should be able to perform the calculation without any working.</p> <p style="text-align: center;">$-5 + 3 = -2$</p>

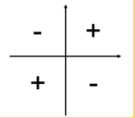
Subtraction

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
Students should be able to use double-sided counters to represent the two numbers and the idea of zero pairs.	Students should draw the manipulatives as a diagram in their books, clearly showing the zero-pairs.	Students should use the ideas in the Physical and Pictorial stages be able to determine, through using 'scaled' double-sided counter diagrams to determine if the answer is positive or negative.	Students should be able to state an answer without the use of a diagram.
Example(s)	Example(s) - These should be seen in books		
<p>Calculate $-6 - -2$ "Negative 6 subtract -2"</p>			
<p>Students should use double-sided counters to represent the -6. https://mathsbot.com/manipulatives/doubleSidedCounters may be useful for this representation.</p> <p>Students should then understand that subtracting -2 is equivalent to +2.</p> <p>Students should then add these counters to their representation.</p>  <p>Students should then identify the zero-pairs in their representation and remove them.</p>  <p>Students should then be able to identify their answer from their remaining counters.</p> $-6 - -2 = -4$	<p>Students should represent the calculation as in the Physical Stage, however, for short it may be beneficial for students to use - and + signs instead of drawing full circles.</p> $\begin{array}{cccccc} - & - & - & - & - & - \\ + & + & & & & & \end{array}$ <p>Students should then identify the zero-pairs in their representation and circle them. They could show they have been removed by crossing through them</p>  <p>Students should then be able to identify their answer from their remaining symbols.</p> $-6 - -2 = -4$	<p>Students should represent this by drawing a larger negative circle subtract a smaller negative circle. https://mathsbot.com/manipulatives/directedCounters may be useful for this representation.</p> <p>For weaker students a number line representation in addition may also be beneficial.</p>  <p>Students should then apply the additive inverse as they have done in previous stages and re-draw.</p>  <p>Notice the size of the second circle does not change.</p> <p>Students should be able to determine from the calculation that there are will be more negative counters than positive ones in this calculation, therefore the answer must be negative.</p> $-6 - -2 = -4$	<p>Students should be able to perform the calculation without any working.</p> $-6 - -2 = -4$

Multiplication

GENERAL TEACHING & LEARNING POINTS

- Students should be confident using factors and multiples and the use of a co-ordinate grid.
- Students should understand that the opposite side of the x or y axis involves multiplying by -1 (flipping the counters over)



Stage 1 Physical

Students should be able to use **double-sided counters** and a numbered grid to represent the problem as an area model.

Example(s)

Stage 2 Pictorial

Students should draw the manipulatives as a diagram in their books.

Stage 3 Semi-Abstract

Students should use the ideas in the Physical and Pictorial stages to be able to determine an answer, through using the area of a rectangle model.

Stage 4 Abstract

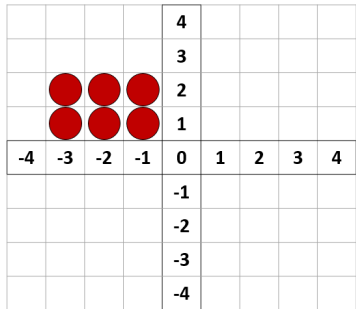
Students should be able to state an answer without the use of a diagram.

Example(s) - These should be seen in books

Calculate -3×2
"Negative 3 multiplied by 2"

For consistency students could represent the first number on the x-axis and the second the y-axis.

Students should use the grid as shown below:

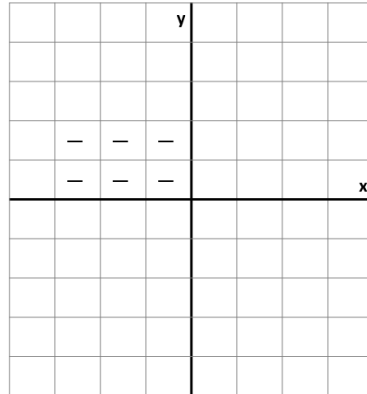


The counters are -3 along the x axis and +2 lots of this (so +2 on the y-axis).

Students should then be able to identify their answer from the counters on their grid

$$-3 \times 2 = -6$$

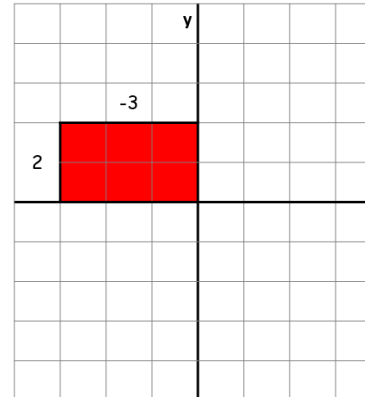
Students should draw a small grid in their exercise books and place either + or - symbols in each square to represent the same idea as in the physical stage.



Students should then be able to identify their answer from the signs in their grid

$$-3 \times 2 = -6$$

Students should draw a similar diagram to those in the Physical and Pictorial stage, however at this stage students should represent the calculation through an area model.



Students should then apply their knowledge of understanding

$$-3 \times 2 = -6$$

Students should be able to perform the calculation without any working.

$$-3 \times 2 = -6$$

Division

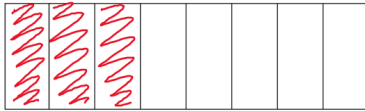
Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
<p>Students should be able to use double-sided counters and a numbered grid to represent the problem as an area model.</p>	<p>Students should draw the manipulatives as a diagram in their books.</p>	<p>Students should use the ideas in the Physical and Pictorial stages be able to determine an answer, through using the area of a rectangle model.</p>	<p>Students should be able to state an answer without the use of a diagram.</p>
Example(s)	Example(s) - These should be seen in books		
<p>Calculate $-8 \div -2$ "Negative 8 divided by negative 2"</p>			
<p>Students will be confident with the + - grid below.</p> <div style="text-align: center;"> </div> <p>Students should know that for this calculation -8 represents the counters required, and this would be represented in quadrant 2 or 4.</p> <p>Student should understand that one dimension needs to be -2.</p> <p>Either representation below could be used.</p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> </div> <div style="text-align: center;"> </div> </div> <p>Students should then be able to identify the solution to the problem is the length/width of the other dimension of the diagram.</p> <p style="text-align: center;">$-8 \div -2 = 4$</p>	<p>Students will be confident with the + - grid below.</p> <div style="text-align: center;"> </div> <p>Students should know that for this calculation -8 represents the counters required, and this would be represented in quadrant 2 or 4.</p> <p>Student should understand that one dimension needs to be -2.</p> <p>Either representation below could be used.</p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> </div> <div style="text-align: center;"> </div> </div> <p>Students should then be able to identify the solution to the problem is the length/width of the other dimension of the diagram.</p> <p style="text-align: center;">$-8 \div -2 = 4$</p>	<p>Students should represent the calculation as an area as with Dividing Numbers.</p> <p>Either representation below could be used.</p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> </div> <div style="text-align: center;"> </div> </div> <p>Students should then be able to identify the solution to the problem is the length/width of the other dimension of the diagram.</p> <p style="text-align: center;">$-8 \div -2 = 4$</p>	<p>At this stage students should be able to perform the calculation without the need for any working.</p> <p style="text-align: center; font-size: 1.2em;">$-8 \div -2 = 4$</p>

EQUIVALENT FRACTIONS

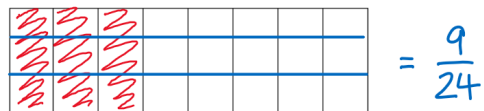
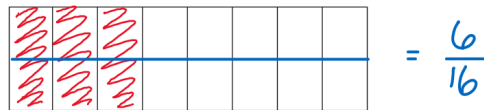
Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
N/A	Students should then be able to represent each fraction as a diagram, one split horizontally and the other vertically	Students should use a scale factor approach to find equivalent fractions.	Students should be able to state equivalent fractions.
Example(s)	Example(s) - These should be seen in books		

Write down three fractions that are equivalent to $\frac{3}{8}$

Students should represent the $\frac{3}{8}$ fraction using a diagram, split either horizontally OR vertically.



Students should then understand that as long as this is **equally** split horizontally (or vertically if the original diagram is split horizontally) then the fractions will be equivalent to $\frac{3}{8}$



This may be added at a later date.

Students should confidently understand that if the numerators and denominators are multiplied or divided by the same factor then the two fractions will be equivalent.

This multiplication should be shown at this stage.

$$\frac{3}{8} \xrightarrow{\times 2} \frac{6}{16}$$

$$\frac{3}{8} \xrightarrow{\times 3} \frac{9}{24}$$

At this stage students should be able to immediately state fractions that are equivalent to $\frac{3}{8}$

CONVERTING BETWEEN MIXED NUMBERS AND IMPROPER FRACTIONS

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract																																																								
N/A	Students should represent the initial quantity using a bar diagram.	Students at this stage should be able to determine the number of parts in the wholes plus any extras.	Students should be able to state the equivalent improper fraction/mixed number without any working.																																																								
Example(s)	Example(s) - These should be seen in books																																																										
Write $\frac{11}{5}$ as a mixed number.																																																											
<p>This may be added at a later date.</p>	<p>Students should understand that, since the denominator is 5, each whole one is split into 5 pieces.</p> <p>Students should draw a diagram to show the 11 parts.</p> <div style="display: flex; align-items: center; justify-content: center;"> <table border="1" style="border-collapse: collapse; text-align: center; margin-right: 10px;"> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr> <tr><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr> <tr><td>11</td><td></td><td></td><td></td><td></td></tr> </table> <div style="margin-left: 10px;"> <p> whole</p> <p> whole</p> <p>$\frac{1}{5}$</p> <p>$= 2 \frac{1}{5}$</p> </div> </div> <p>Students should then identify the whole parts and the fraction parts and combine.</p>	1	2	3	4	5	6	7	8	9	10	11					<p>Students should be encouraged to think of how many whole 5s are in 11, and what the remainder is.</p> $\frac{11}{5} = 2 \text{ remainder } 1$ $= 2 \frac{1}{5}$	<p>At this stage students should be able to immediately state the answer.</p> $\frac{11}{5} = 2 \frac{1}{5}$																																									
1	2	3	4	5																																																							
6	7	8	9	10																																																							
11																																																											
Write $3\frac{2}{7}$ as a mixed number.																																																											
<p>This may be added at a later date.</p>	<p>Students should understand that, since the denominator is 7, each whole one is split into 7 pieces.</p> <p>Students should then count the number of parts.</p> <div style="display: flex; align-items: center; justify-content: center;"> <div style="margin-right: 10px;"> <p>3 {</p> <p>2/7 {</p> </div> <table border="1" style="border-collapse: collapse; text-align: center;"> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr> <tr><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td><td>13</td><td>14</td></tr> <tr><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td><td>21</td></tr> <tr><td>22</td><td>23</td><td></td><td></td><td></td><td></td><td></td></tr> </table> <div style="margin-left: 10px;"> <p>$\frac{23}{7}$</p> </div> </div>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23						<p>Students should partly rely on the diagram at this stage.</p> <p>Students should be able to determine the number of sevenths in the three whole ones and add the extras.</p> <div style="display: flex; align-items: center; justify-content: center;"> <div style="margin-right: 10px;"> <p>3 {</p> <p>2/7 {</p> </div> <table border="1" style="border-collapse: collapse; text-align: center;"> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr> <tr><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td><td>13</td><td>14</td></tr> <tr><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td><td>21</td></tr> <tr><td>22</td><td>23</td><td></td><td></td><td></td><td></td><td></td></tr> </table> <div style="margin-left: 10px;"> <p>$3 \times 7 = 21$</p> <p>$21 + 2 = 23$</p> <p>$= \frac{23}{7}$</p> </div> </div>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23						$3\frac{2}{7} = \frac{23}{7}$
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ADDING AND SUBTRACTING FRACTIONS


Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
N/A	Students should then be able to represent each fraction as a diagram, one split horizontally and the other vertically	Students should be able to determine the LCM of the two denominators.	Students should be able to add and subtract fractions without the need for a diagram; understanding the need for the LCM.
Example(s)	Example(s) - These should be seen in books		

Calculate $\frac{2}{5} + \frac{1}{3}$

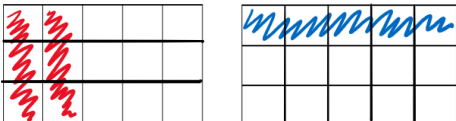
This may be added at a later date.

Students should represent each of the two fractions as separate diagrams.

ONE DIAGRAM SHOULD BE SPLIT VERTICALLY AND THE OTHER HORIZONTALLY.

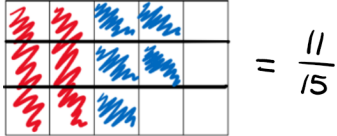


Students should then identify the LCM in order to find a common denominator and split each representation into this number of parts - See Equivalent Fractions.




Students should then understand that now all the small parts in the diagram are the same size, these can be combined.

For addition, students should add the parts in both diagrams to a single diagram.




[For subtraction, students should cross through a part in the first diagram for each part in the second diagram]

$\frac{2}{5} - \frac{1}{3}$



Students should be able to determine the LCM of the two denominators, 15 in this case.

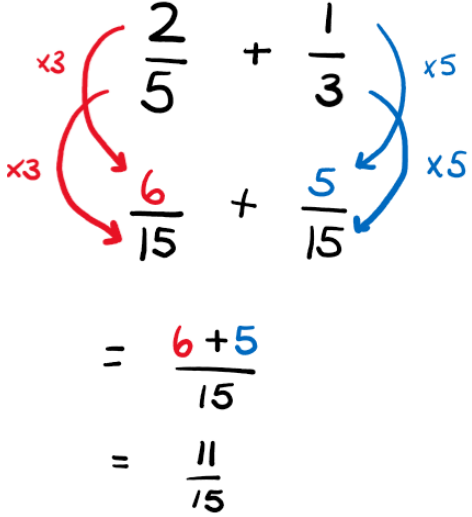
Students should then draw diagrams to represent these.



$\frac{2}{5} = \frac{6}{15}$ $\frac{1}{3} = \frac{5}{15}$

$\frac{6}{15} + \frac{5}{15}$

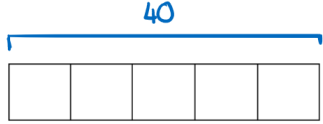
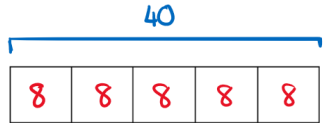
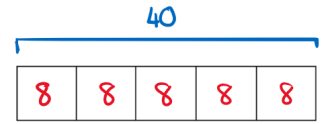
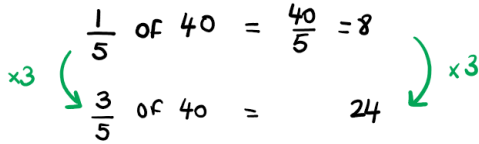
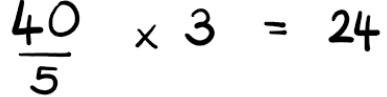
$= \frac{11}{15}$



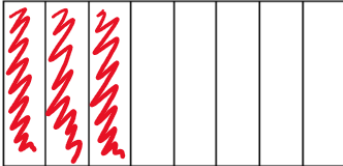
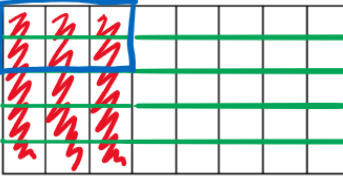
$= \frac{6+5}{15}$

$= \frac{11}{15}$

FRACTIONS OF AMOUNTS

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
N/A	Students should be able to represent the problem using a bar model.	Students should use a proportion idea first identifying a unit fraction and then scaling up.	Students should use a simple written method that shows finding a unit fraction and then multiplying.
Example(s)	Example(s) - These should be seen in books		
Find $\frac{3}{5}$ of 40			
<p>This may be added at a later date.</p>	<p>Students should represent the problem as a bar model, splitting the whole bar into the number of parts shown in the denominator and clearly showing the total.</p> <div style="text-align: center;">  </div> <p>Students should then work out the value of one part, students should then add this to their diagram.</p> <div style="text-align: center;">  </div> <p style="text-align: center;">$\frac{1}{5}$ of 40 = $\frac{40}{5}$ = 8</p> <p>Students should then clearly show how they have obtained the $\frac{3}{5}$</p> <div style="text-align: center;">  </div> <p style="text-align: center;">$\frac{1}{5}$ of 40 = $\frac{40}{5}$ = 8</p> <p style="text-align: center;">$\frac{3}{5}$ of 40 = 8 x 3 = 24</p>	<p>Students should find $\frac{1}{5}$ and then multiply this by 3.</p> <div style="text-align: center;">  </div>	<p>Students should show find $\frac{1}{5}$ and then multiply this by 3 as a single calculation.</p> <div style="text-align: center;">  </div>

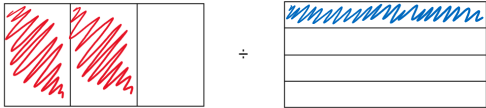

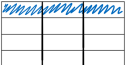
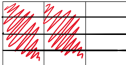
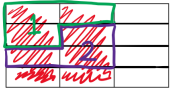
MULTIPLYING FRACTIONS

GENERAL TEACHING & LEARNING POINTS	<ul style="list-style-type: none"> To be consistent with diagrams, students should understand that multiplying fractions means the second a fraction OF the first, e.g. $\frac{1}{6} \times \frac{1}{2}$ means $\frac{1}{2}$ of $\frac{1}{6}$ 		
Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
N/A	Students should be able to represent the problem using a bar model.	Students should clearly show their written method, simplifying their answer.	Students at this stage should look for common factors to cancel to avoid larger numerators and denominators.
Example(s)	Example(s) - These should be seen in books		
Calculate $\frac{3}{8} \times \frac{2}{5}$			
<p style="text-align: center;">This may be added at a later date.</p>	<p>Students should draw a diagram to represent the $\frac{3}{8}$</p>  <p>Students should then find $\frac{2}{5}$ of the shaded part. Students should divide this into 5 part.</p>  <p>Students should circle the $\frac{2}{5}$</p> $= \frac{6}{40}$ $= \frac{3}{20}$	<p>Students should complete the calculation without the need for a diagram. It is important that students include the $\frac{3 \times 2}{8 \times 5}$ step to clearly show their understanding.</p> $\frac{3}{8} \times \frac{2}{5}$ $= \frac{3 \times 2}{8 \times 5}$ $= \frac{6}{40}$ $= \frac{3}{20}$	<p>Students should be encouraged to spot possible calculations after the first step, as shown below.</p> $\frac{3}{8} \times \frac{2}{5}$ $= \frac{3 \times \cancel{2}^1}{\cancel{4}^1 \times 5}$ $= \frac{3 \times 1}{4 \times 5}$ $= \frac{3}{20}$

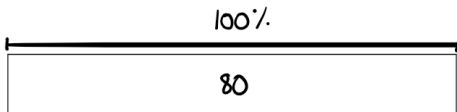

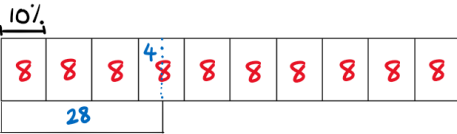
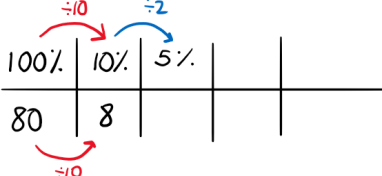
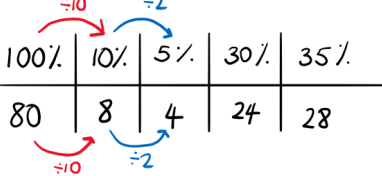
DIVIDING FRACTIONS

GENERAL TEACHING & LEARNING POINTS	<ul style="list-style-type: none"> "Keep Flip Change" should never be used Students should be encouraged to "multiply by the reciprocal" Students should be encouraged to think of $\frac{2}{3} \div \frac{1}{4}$ as "How many $\frac{1}{4}$ are there in $\frac{2}{3}$" 		
Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
N/A	Students should be able to represent the problem using a bar model.	Students should clearly show their written method, simplifying their answer.	Students at this stage should clearly show division by a fraction is equivalent to multiplying by its reciprocal.
Example(s)	Example(s) - These should be seen in books		

Calculate $\frac{2}{3} \div \frac{1}{4}$

<p>This may be added at a later date.</p>	<p>Students should represent each fraction in the calculation as separate diagrams, splitting one vertically and the other horizontally.</p>  <p>Students should use their knowledge of Equivalent Fractions to split their diagrams so that the parts in each diagram are equal sizes.</p>  <p>Students should understand that students need to determine</p> <p>how many  in </p> <p>Students should then circle the groups of three parts</p>  <p style="text-align: center;">2 Whole $\frac{2}{3}$ left</p> <p style="text-align: center;">$= 2\frac{2}{3}$</p>	<p>Students should understand that if they multiply both fractions by the reciprocal of the second, this reduces the calculation to a multiplication, and division by 1.</p> $\frac{2}{3} \div \frac{1}{4}$ $\overset{\times \frac{4}{1}}{\left(\frac{2}{3} \times \frac{4}{1}\right)} \div \overset{\times \frac{4}{1}}{\left(\frac{1}{4} \times \frac{4}{1}\right)}$ $= \frac{8}{3} \div 1$ $= \frac{8}{3}$ $= 2\frac{2}{3}$	<p>Students should represent this multiplication by the reciprocal as a single calculation.</p> $\frac{2}{3} \div \frac{1}{4}$ $= \frac{2}{3} \times \frac{4}{1}$ $= \frac{8}{3}$ $= 2\frac{2}{3}$
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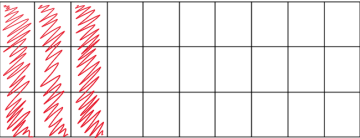

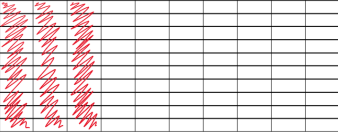
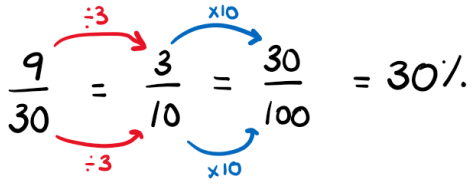
PERCENTAGE OF AMOUNTS

GENERAL TEACHING & LEARNING POINTS													
Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract										
N/A	Students should be able to represent the problem using a bar model.	Students should use a written method, set out using a proportion table .	Students should use a written, multiplier , method in order to calculate the required percentage.										
Example(s)	Example(s) - These should be seen in books												
Calculate 35% of 80													
<p>This may be added at a later date.</p>	<p>Students should use a bar model to represent the initial 100%.</p> <div style="text-align: center;">  </div> <p>Students should then split their bar model into a suitable number of parts.</p> <div style="text-align: center;">  </div> <p style="text-align: center;">$\frac{80}{10} = 8$</p> <p>Students should then determine the correct percentage by using a second bar underneath</p> <div style="text-align: center;">  </div> <p style="text-align: center;">$\frac{80}{10} = 8$</p>	<p>Students should represent the initial amount at 100% in a proportion table.</p> <div style="text-align: center;"> <table border="1" style="margin: auto;"> <tr> <td style="padding: 5px;">100%</td> <td style="width: 20px;"></td> <td style="width: 20px;"></td> <td style="width: 20px;"></td> <td style="width: 20px;"></td> </tr> <tr> <td style="padding: 5px;">80</td> <td></td> <td></td> <td></td> <td></td> </tr> </table> </div> <p>Students should then calculate a suitable percentage(s) to use to calculate the final amount, (10% and 5% in this case).</p> <div style="text-align: center;">  </div> <p>Students should complete their proportion table to obtain the required final percentage.</p> <p>Students should clearly state their final answer.</p> <div style="text-align: center;">  </div> <p style="text-align: center;">$35\% \text{ of } 80 = 28$</p>	100%					80					<p>Students should understand that 'of' means multiply and determine the multiplier.</p> <div style="text-align: center; margin-top: 20px;"> $0.35 \times 80 = 28$ </div>
100%													
80													

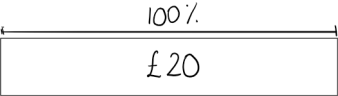

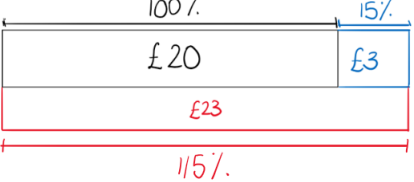
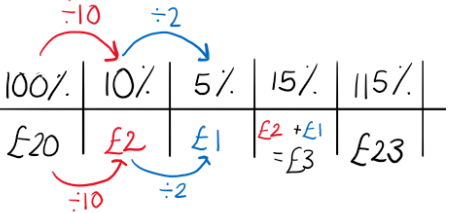
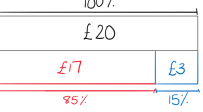
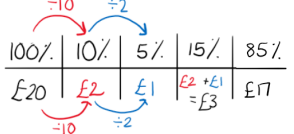
ONE QUANTITY AS A PERCENTAGE OF ANOTHER

GENERAL TEACHING & LEARNING POINTS			
Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
N/A	Students should be able to represent the problem using a bar model.	Students should use a written method using equivalent fractions.	Students should understand that they can write their calculation as a single line of working, which could be entered into a calculator if permitted.
Example(s)	Example(s) - These should be seen in books		

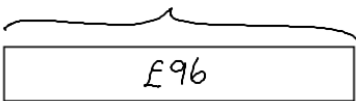
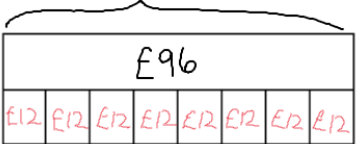
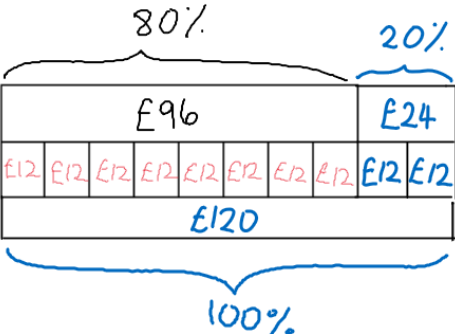
Zuzanna scored 9 out of 30 on a maths test. Work out her score as a percentage.

<p>This may be added at a later date.</p>	<p>Students should represent the problem as a diagram.</p> <p>They should divide the total amount into an appropriate amount, usually a factor of 100.</p> <div style="text-align: center;">  $\frac{9}{30}$ </div> <p>Students should understand that percentage means 'per 100'. They should then split their diagram into 100 parts. It may be appropriate to split their diagram using two stages.</p> <div style="text-align: center;">  $\frac{3}{10}$ </div> <p style="text-align: center;">Followed by</p> <div style="text-align: center;">  $\frac{30}{100} = 30\%$ </div>	<p>Students should use their understanding obtained in the Pictorial Stage and the idea that this relates back to equivalent fractions, to show their working as below:</p> <div style="text-align: center;">  $\frac{9}{30} = \frac{3}{10} = \frac{30}{100} = 30\%$ </div>	$\frac{9}{30} \times 100 = 30\%$
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PERCENTAGE INCREASE/DECREASE

GENERAL TEACHING & LEARNING POINTS	<ul style="list-style-type: none"> Students should understand that increasing (or decreasing) by a percentage is equivalent to finding a percentage of an amount above (or below) 100%. 		
Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
N/A	Students should be able to represent the problem using a bar model.	Students should use a proportion table to find the required percentage.	Students should be able to use a calculator, through a multiplier, to determine the final amount.
Example(s)	Example(s) - These should be seen in books		
<p>Increase £20 by 15%</p>			
<p>This may be added at a later date.</p>	<p>Students should use a bar model to represent the initial 100%.</p> <div style="text-align: center;">  <p>100% £20</p> </div> <p>Students should then use the method shown in Finding Percentage of an Amount, to find the percentage to increase (or decrease) by.</p> <div style="text-align: center;">  <p>$\frac{£30}{10} = £2$</p> </div> <p>Students should then add on the £3 to the end of their diagram, clearly labelling this as 115%.</p> <div style="text-align: center;">  <p>100% 15% £20 £3 £23 115%</p> </div>	<p>Students should use their understanding of proportionality to set their working out in a proportion table, as shown below.</p> <div style="text-align: center;">  <p> $\begin{array}{c c c c c} \div 10 & \div 2 & & & \\ \hline 100\% & 10\% & 5\% & 15\% & 115\% \\ \hline £20 & £2 & £1 & £2 + £1 & £23 \\ \hline & \div 10 & \div 2 & = £3 & \end{array}$ </p> </div>	<p>Students should understand that 'of' means multiply and determine the multiplier.</p> <div style="text-align: center; font-size: 1.2em;"> $1.15 \times 20 = £23$ </div>
	<p style="text-align: center;">Decrease £20 by 15%</p> <div style="text-align: center;">  <p>100% £20 £17 £3 85% 15%</p> </div>	<p style="text-align: center;">Decrease £20 by 15%</p> <div style="text-align: center;">  <p> $\begin{array}{c c c c c} \div 10 & \div 2 & & & \\ \hline 100\% & 10\% & 5\% & 15\% & 85\% \\ \hline £20 & £2 & £1 & £2 + £1 & £17 \\ \hline & \div 10 & \div 2 & = £3 & \end{array}$ </p> </div>	<p style="text-align: center;">Decrease £20 by 15%</p> <div style="text-align: center; font-size: 1.2em;"> $0.85 \times 20 = £17$ </div>

REVERSE PERCENTAGES

GENERAL TEACHING & LEARNING POINTS	<ul style="list-style-type: none"> Students should understand that increasing (or decreasing) by a percentage is equivalent to finding a percentage of an amount above (or below) 100%. 		
Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
N/A	Students should be able to represent the problem using a bar model.	Students should use a proportion table to find the required percentage.	Students should be able to use a calculator, through a multiplier, to determine the final amount.
Example(s)	Example(s) - These should be seen in books		
A shop has a 20% off sale. A pair of trainers have a sale price of £96. How much were the trainers before the sale?			
<p>This may be added at a later date.</p>	<p>Students should use a bar model to represent the discounted amount.</p> <div style="text-align: center;"> 80%  </div> <p>Students should then use the method shown in Finding Percentage of an Amount, to find the value of 100%.</p> <div style="text-align: center;"> 80%  </div> <div style="text-align: center;"> 80% 20%  </div>	<p>Students should use their understanding of proportionality to set their working out in a proportion table, as shown below.</p> <div style="text-align: center;"> $\begin{array}{c c c} \div 8 & \times 10 & \\ \hline 80\% & 10\% & 100\% \\ \hline \text{£}96 & \text{£}12 & \text{£}120 \end{array}$ <p>Original Price = £120</p> </div>	$\begin{aligned} 80\% \text{ of Original Amount} &= \text{£}96 \\ 0.8 \times \text{original Amount} &= \text{£}96 \\ \text{Original Amount} &= \frac{\text{£}96}{0.8} \\ &= \text{£}120 \end{aligned}$ <p>(The first line of working could be omitted here)</p>

CONVERTING UNITS

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
Students could use Cuisenaire Rods (base 10) to represent the original length.	Students should use a bar model to represent a similar idea to that of the Physical Stage.	Students should use a proportion table to scale up or down each quantity.	Students should be able to state the final conversion without any working.
Example(s)	Example(s) - These should be seen in books		

Convert 2.4m into cm.

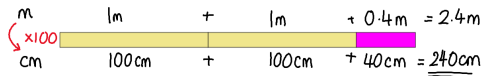
It will be beneficial if students use the manipulatives on top of a whiteboard for this stage.

Students should represent the 2.4m using two '10' rods and one '4' rod.



Notice the scale factor has been indicated on the left hand side.

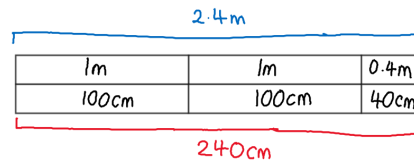
Students should then sum above and below.



Students should then state their final answer.

$$2.4\text{ m} = 240\text{ cm}$$

Students should represent the problem as a bar model clearly equating each part of the bar model in both units, as shown below.



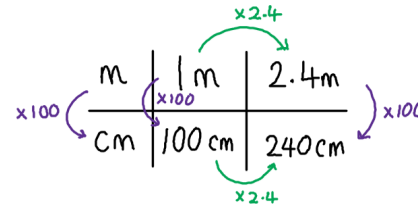
Notice, the diagram is roughly to scale (0.4m is smaller than the 1m), this emphasises that, when converted, the 0.4m cannot be more than 100cm.

Students should then state their final answer.

$$2.4\text{ m} = 240\text{ cm}$$

Students should set the work out in a proportion table.

ALL students should be able to perform the conversion in two different ways, shown in green and purple below.




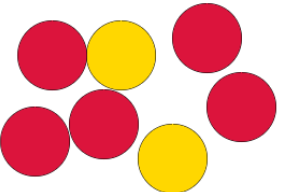
Students should then state their final answer.

$$2.4\text{ m} = 240\text{ cm}$$


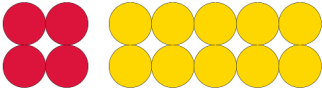

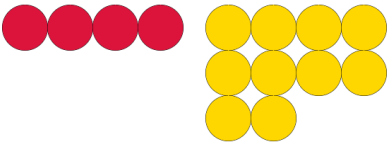
Students will be able to state the conversion immediately.

$$2.4\text{ m} = 240\text{ cm}$$

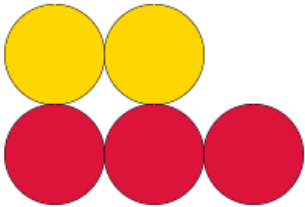

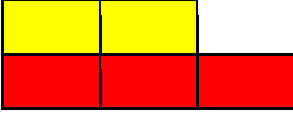
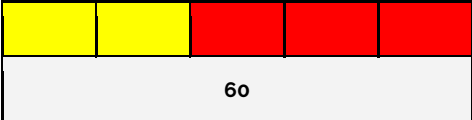
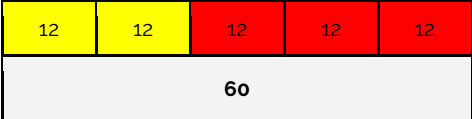
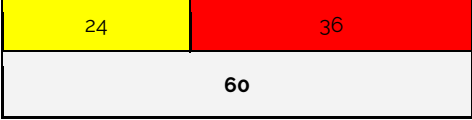
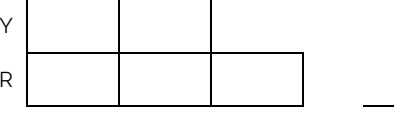
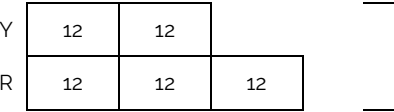
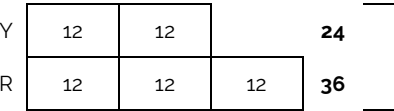
WRITING RATIOS AS A FRACTIONS

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract																																												
Students should be able to represent a problem using counters.	Students should be able to represent the information using a bar model.	Students should be able to use a single bar model to aid their understanding	Students should be able to write the fraction without the need for a diagram.																																												
Example(s)	Example(s) - These should be seen in books																																														
The ratio of yellow to red counters is 2:5. What fraction of the counters is yellow?																																															
<p>Students should be able to represent the problem as below.</p>  <p>Notice that this clearly shows the different counters separately.</p> <p>Students should then physically move the counters together, this is important so that students understand that in total there are 7 parts.</p>  <p>Students should then apply their understanding of what a fraction is to identify that 5 out of the 7 counters are red. Students should clearly write this.</p> <p style="font-size: 1.2em;">2 out of 7 are yellow</p> <p style="font-size: 1.5em;">$\frac{2}{7}$ are yellow</p>	<p>Students should represent this as a bar model, with colour side-by-side.</p> <table border="1" style="margin: 0 auto; text-align: center;"> <tr> <td>Y</td><td>Y</td><td>:</td><td>R</td><td>R</td><td>R</td><td>R</td><td>R</td> </tr> </table> <p>Students should then combine each side into a single bar to represent the total number of parts.</p> <table border="1" style="margin: 0 auto; text-align: center;"> <tr> <td>Y</td><td>Y</td><td>R</td><td>R</td><td>R</td><td>R</td><td>R</td> </tr> </table> <p>Students should then add the fraction of each colour underneath their representation.</p> <table border="1" style="margin: 0 auto; text-align: center;"> <tr> <td>Y</td><td>Y</td><td>R</td><td>R</td><td>R</td><td>R</td><td>R</td> </tr> <tr> <td colspan="2" style="border: none;">$\frac{2}{7}$</td> <td colspan="5" style="border: none;">$\frac{5}{7}$</td> </tr> </table> <p style="text-align: center; font-size: 1.2em;">$\frac{2}{7}$ are yellow</p>	Y	Y	:	R	R	R	R	R	Y	Y	R	R	R	R	R	Y	Y	R	R	R	R	R	$\frac{2}{7}$		$\frac{5}{7}$					<p>Students should understand the idea of a ratio by this stage.</p> <p>Students should therefore be able to draw the diagram below:</p> <table border="1" style="margin: 0 auto; text-align: center;"> <tr> <td>Y</td><td>Y</td><td>R</td><td>R</td><td>R</td><td>R</td><td>R</td> </tr> </table> <p>Students should then be able to state the appropriate fraction from this diagram alone.</p> <p style="font-size: 1.5em; text-align: center;">$\frac{2}{7}$ are yellow</p>	Y	Y	R	R	R	R	R	<p>Students should be able to represent the problem using a table. Consistency is important here as this layout will be used regularly during sharing in a ratio.</p> <table style="margin: 0 auto; text-align: center;"> <tr> <td>Yellow</td><td>:</td><td>Red</td><td style="border-left: 1px solid black; padding-left: 10px;">Total Parts</td> </tr> <tr> <td>2</td><td>:</td><td>5</td><td style="border-left: 1px solid black; padding-left: 10px;">$2 + 5 = 7$</td> </tr> </table> <p style="text-align: center; font-size: 1.5em;">$\frac{2}{7}$ are yellow</p>	Yellow	:	Red	Total Parts	2	:	5	$2 + 5 = 7$
Y	Y	:	R	R	R	R	R																																								
Y	Y	R	R	R	R	R																																									
Y	Y	R	R	R	R	R																																									
$\frac{2}{7}$		$\frac{5}{7}$																																													
Y	Y	R	R	R	R	R																																									
Yellow	:	Red	Total Parts																																												
2	:	5	$2 + 5 = 7$																																												

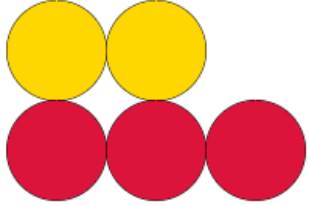
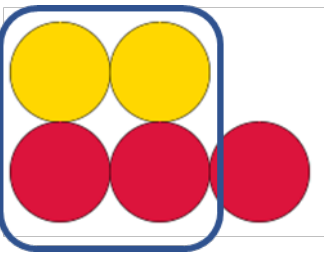

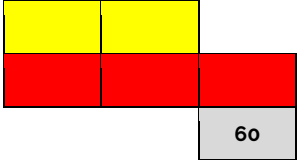
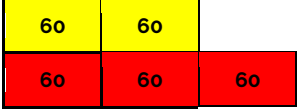
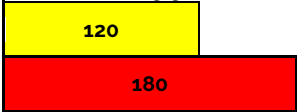
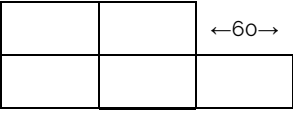
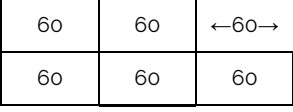
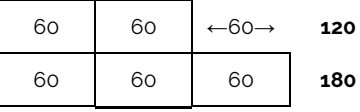
SIMPLIFYING RATIO

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract																																																												
Students should be able to represent a problem using manipulatives (counters or multi-link cubes).	Students should be able to represent the information using a 'comparative' bar model and a 'cumulative' bar model.	Students should use a 'comparative' bar model and use abstract knowledge that each part is of equal value, thus using division.	Students should be able to share the ratio without needing a diagram. Students can then use equality of ratios and proportions.																																																												
Example(s)	Example(s) - These should be seen in books																																																														
Simplify the ratio 4:10.																																																															
<p>Students should be able to represent the ratio using counters.</p>  <p>Students should then attempt to re-arrange the counters into the same number of rows, two in this case.</p>  <p>Students should apply their understanding of what a ratio is to remove all rows of counters except the first.</p>  <p>Students should then be able to state that 4:10 can be simplified to 2:5.</p>	<p>Students should represent this as a 'comparative' bar model, with colour on top of each other</p> <table border="1" style="margin: 5px auto; border-collapse: collapse;"> <tr><td>Y</td><td>Y</td><td>Y</td><td>Y</td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td>R</td><td>R</td><td>R</td><td>R</td><td>R</td><td>R</td><td>R</td><td>R</td><td>R</td><td>R</td></tr> </table> <p>Students should then recognise that the diagram would be equal if two yellow parts were moved as shown:</p> <table border="1" style="margin: 5px auto; border-collapse: collapse;"> <tr><td>Y</td><td>Y</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td>R</td><td>R</td><td>R</td><td>R</td><td>R</td><td></td><td></td><td></td><td></td><td></td></tr> </table> <p>Students should apply their understanding of what a ratio is to remove all but one group and the ratio remains equal.</p> <table border="1" style="margin: 5px auto; border-collapse: collapse;"> <tr><td>Y</td><td>Y</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td>R</td><td>R</td><td>R</td><td>R</td><td>R</td><td></td><td></td><td></td><td></td><td></td></tr> </table> <p style="text-align: center;">$R : Y$ $= 2 : 5$</p>	Y	Y	Y	Y							R	R	R	R	R	R	R	R	R	R	Y	Y									R	R	R	R	R						Y	Y									R	R	R	R	R						<p>Students should represent the ratio in a single line.</p> <p style="text-align: center; font-size: 1.2em;">Y Y Y Y : R R R R R R R R R R</p> <p>Students should then be able to identify that each side of the ratio could be grouped into 2s. Higher attaining students could identify that this is the highest common factor of the two parts.</p> <p style="text-align: center;">Students should circle these groups.</p> <p style="text-align: center;">YYYY : RRRRRRRRRR</p> <p>Students should then understand the ratio of the original number of each colour is the same ratio as the number of groups.</p> <p style="text-align: center; font-size: 1.5em;">$R : Y$ $= 2 : 5$</p>	<p>Students should represent the ratio in a table.</p> <p>Students should clearly show that the division operations need to be applied to both sides of the ratio to keep them equivalent.</p> <p style="text-align: center; font-size: 1.5em;">$R : Y$ $4 : 10$ $\div 2 \quad 2 : 5 \quad \div 2$</p>
Y	Y	Y	Y																																																												
R	R	R	R	R	R	R	R	R	R																																																						
Y	Y																																																														
R	R	R	R	R																																																											
Y	Y																																																														
R	R	R	R	R																																																											
<p style="text-align: center;">Discussion Point</p> <p>Students should be able to confidently explain why the counters cannot be arranged as below</p> 																																																															

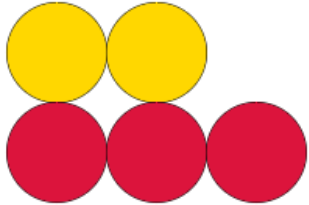
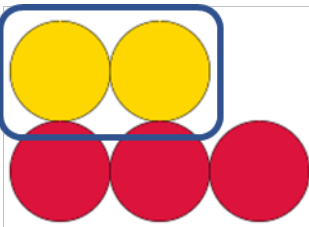
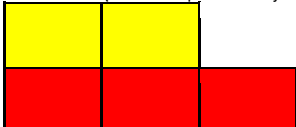
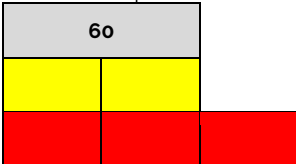
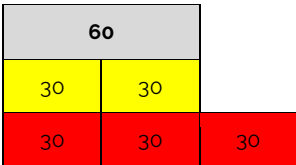
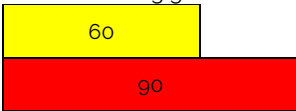
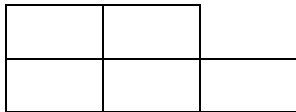
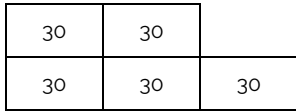
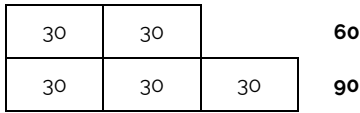
SHARING IN A RATIO – SHARING TOTAL

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract																
Students should be able to represent a problem using manipulatives (counters or multi-link cubes).	Students should be able to represent the information using a 'comparative' bar model and a 'cumulative' bar model.	Students should use a 'comparative' bar model and use abstract knowledge that each part is of equal value, thus using division.	Students should be able to share the ratio without needing a diagram. Students can then use equality of ratios and proportions.																
Example(s)	Example(s) - These should be seen in books																		
There are 60 counters in a box. They are either yellow or red. The ratio of yellow counters to red counters is 2:3. How many of each colour are in the box?																			
<p>Students should be able to represent the problem as below.</p>  <p>Students should understand that the total value of all five parts is 60 and all parts represent the same amount, i.e. the manipulative below represents 60.</p> <p>Students should then identify the value of one cube:</p>  <p>Students should then understand since there are 2 yellow and 3 red blocks this represents: Yellow: $12 + 12 = 24$ Red: $12 + 12 + 12 = 36$</p> <p>[Notice that this is represented as repeated addition and not multiplication]</p> <p style="text-align: center;">So there are 24 yellow and 36 red.</p>	<p>Students should be able to represent the information in the question pictorially as a 'comparative' bar model as below:</p>  <p>[Notice this is represented as yellow above red, this makes it easier to compare the quantities and also for more difficult problems - see later]</p> <p>Students should understand that the total value of all five parts is 60 and represent this as a 'cumulative' bar model.</p>  <p>Students should then be able to identify the value of each part.</p>  <p>Students will then see the number of each colour.</p>  <p style="text-align: center;">So there are 24 yellow and 36 red.</p>	<p>Students should be able to represent the information in the question pictorially, including the total number, as below:</p>  <p>Students should then know to find the value of each part is the total divided by the number of parts.</p> <p>Students should then annotate the value of each part on their diagram:</p>  <p style="text-align: center;">Totalling each colour</p>  <p style="text-align: center;">So there are 24 yellow and 36 red.</p>	<p>Students should be able to identify the <u>total</u> number of parts, then identify the scale factor to multiply to make the total parts to the total number of counters in this case.</p> <p>Note: Parts in <u>black</u> are step one, parts in <u>blue</u> are step two.</p> <table style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <tr> <td style="padding: 0 10px;">Yellow</td> <td style="padding: 0 10px;">:</td> <td style="padding: 0 10px;">Red</td> <td style="border-left: 1px solid black; padding: 0 10px;">Total Parts</td> </tr> <tr> <td style="text-align: center; padding: 0 10px;">2</td> <td></td> <td style="text-align: center; padding: 0 10px;">3</td> <td style="border-left: 1px solid black; text-align: center; padding: 0 10px;">5</td> </tr> <tr> <td style="text-align: center; padding: 0 10px;"><i>X12</i></td> <td></td> <td style="text-align: center; padding: 0 10px;"><i>X12</i></td> <td style="border-left: 1px solid black; text-align: center; padding: 0 10px;"><i>X12</i></td> </tr> <tr> <td style="text-align: center; padding: 0 10px; color: blue;">24</td> <td></td> <td style="text-align: center; padding: 0 10px; color: blue;">36</td> <td style="border-left: 1px solid black; text-align: center; padding: 0 10px;">60</td> </tr> </table> <p>Students should then clearly state the answer either as a ratio or writing the number of each colour in this case. Which of these will depend on the exact wording of each question.</p> <p style="text-align: center;">Y:R = 24:60</p> <p style="text-align: center;">OR</p> <p style="text-align: center;">Yellow = 24 counters Red = 36 counters</p>	Yellow	:	Red	Total Parts	2		3	5	<i>X12</i>		<i>X12</i>	<i>X12</i>	24		36	60
Yellow	:	Red	Total Parts																
2		3	5																
<i>X12</i>		<i>X12</i>	<i>X12</i>																
24		36	60																



SHARING IN A RATIO - DIFFERENCES

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract												
Students should be able to represent a problem using manipulatives (counters or multi-link cubes).	Students should be able to represent the information using a 'comparative' bar model and a 'cumulative' bar model.	Students should use a 'comparative' bar model and use abstract knowledge that each part is of equal value, thus using division.	Students should be able to share the ratio without needing a diagram. Students can then use equality of ratios and proportions.												
Example(s)	Example(s) - These should be seen in books														
They are either yellow or red counters in a box. The ratio of yellow counters to red counters is 2:3. There are 60 more red counters than yellow. How many yellow counters and red counters were in the box?															
<p>Students should be able to represent the problem as below.</p>  <p>Students should understand that the four blocks circled have equal parts and the extra red block represents the 60 extra red counters.</p>  <p style="text-align: center;">So each block represents 60</p> <p>Students should then understand since there are 2 yellow and 3 red blocks this represents: Yellow: $60 + 60 = 120$ Red: $60 + 60 + 60 = 180$</p> <p style="text-align: center;">So there are 120 yellow and 180 red.</p>	<p>Students should be able to represent the information in the question pictorially as below:</p>  <p>[Notice this is represented as yellow above red, this makes it easier to compare the quantities and also for more difficult problems - see later]</p> <p>Students should understand that the total value of all five parts is 60 and represent this as a bar model.</p>  <p>Students should then be able to identify the value of each part.</p>  <p style="text-align: center;">Totalling gives</p>  <p style="text-align: center;">So there are 120 yellow and 180 red.</p>	<p>Students should be able to represent the information in the question pictorially, including the difference, as below:</p>  <p>Students should then understand each part must represent the same value. Students should annotate their diagram.</p>  <p>Students should then be able to total each part.</p>  <p style="text-align: center;">So there are 120 yellow and 180 red.</p>	<p>Students should be able to identify the <u>difference</u> in the number of parts, then divide to determine the amount represented by 1 part.</p> <p style="text-align: center;"><i>Note: Parts in black are step one, parts in blue are step two.</i></p> <table style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <tr> <td style="text-align: center;">Yellow</td> <td style="text-align: center;">:</td> <td style="text-align: center;">Red</td> <td style="border-left: 1px solid black; text-align: center;">Parts Difference</td> </tr> <tr> <td style="text-align: center;">2</td> <td></td> <td style="text-align: center;">3</td> <td style="border-left: 1px solid black; text-align: center;">1</td> </tr> <tr> <td style="text-align: center;"><i>X60</i> 120</td> <td></td> <td style="text-align: center;"><i>X60</i> 180</td> <td style="border-left: 1px solid black; text-align: center;">60 <i>X60</i></td> </tr> </table> <p>Students should then clearly state the answer either as a ratio or writing the number of each colour in this case. Which of these will depend on the exact wording of each question.</p> <p style="text-align: center;">Y:R = 120:180</p> <p style="text-align: center;">OR</p> <p style="text-align: center;">Yellow = 120 counters Red = 180 counters</p>	Yellow	:	Red	Parts Difference	2		3	1	<i>X60</i> 120		<i>X60</i> 180	60 <i>X60</i>
Yellow	:	Red	Parts Difference												
2		3	1												
<i>X60</i> 120		<i>X60</i> 180	60 <i>X60</i>												

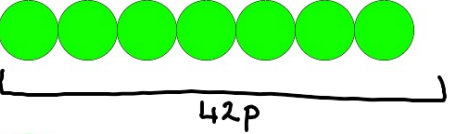
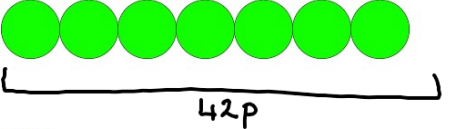


SHARING IN A RATIO – GIVEN ONE AMOUNT

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract												
Students should be able to represent a problem using manipulatives (counters or multi-link cubes).	Students should be able to represent the information using a 'comparative' bar model and a 'cumulative' bar model.	Students should use a 'comparative' bar model and use abstract knowledge that each part is of equal value, thus using division.	Students should be able to share the ratio without needing a diagram. Students can then use equality of ratios and proportions.												
Example(s)	Example(s) - These should be seen in books														
<p>They are either yellow or red counters in a box. The ratio of yellow counters to red counters is 2:3. There are 60 yellow counters. How many counters are in the box in total?</p>															
<p>Students should be able to represent the problem as below.</p>  <p>Students should understand that the yellow blocks represent 60.</p>  <p>So each block represents 30.</p> <p>Students should then understand that the 3 red blocks this represents:</p> <p style="text-align: center;">Red: $30 + 30 + 30 = 90$</p> <p style="text-align: center;">So there are $60 + 90 = 150$ in total.</p>	<p>Students should be able to represent the information in the question pictorially as below:</p>  <p>[Notice this is represented as yellow above red, this makes it easier to compare the quantities and also for more difficult problems - see later!]</p> <p>Students should understand that the total value of all five parts is 60 and represent this as a bar model.</p>  <p>Students should then be able to identify the value of each part.</p>  <p style="text-align: center;">Totalling gives</p>  <p style="text-align: center;">$60+90 = 150$ counters in total</p>	<p>Students should be able to represent the information in the question pictorially, including the total yellow counters., as below:</p>  <p>Students should then understand each part must represent the same value. Students should annotate their diagram.</p>  <p>Students should then be able to total each part and the overall total.</p>  <p style="text-align: center;">$60+90 = 150$ counters in total</p>	<p>Students should be able to identify the number of parts represented by the given quantity, then divide to determine the amount represented by 1 part.</p> <p style="text-align: center;"><i>Note: Parts in black are step one, parts in blue are step two.</i></p> <table style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <tr> <td style="padding: 0 10px;">Yellow</td> <td style="padding: 0 10px;">:</td> <td style="padding: 0 10px;">Red</td> <td style="border-left: 1px solid black; padding: 0 10px;">Total Parts</td> </tr> <tr> <td style="text-align: center; padding: 0 10px;">2</td> <td style="text-align: center; padding: 0 10px;">:</td> <td style="text-align: center; padding: 0 10px;">3</td> <td style="border-left: 1px solid black; text-align: center; padding: 0 10px;">5</td> </tr> <tr> <td style="text-align: center; padding: 0 10px;">$\times 30$ 60</td> <td style="text-align: center; padding: 0 10px;">:</td> <td style="text-align: center; padding: 0 10px;">$\times 30$ 90</td> <td style="border-left: 1px solid black; text-align: center; padding: 0 10px;">$\times 30$ 150</td> </tr> </table> <p>Students should then clearly state the answer either as a ratio or writing the number of each colour in this case. Which of these will depend on the exact wording of each question.</p> <p style="text-align: center;">Y:R = 60:90</p> <p style="text-align: center;">Total = $60 + 90 = 150$</p>	Yellow	:	Red	Total Parts	2	:	3	5	$\times 30$ 60	:	$\times 30$ 90	$\times 30$ 150
Yellow	:	Red	Total Parts												
2	:	3	5												
$\times 30$ 60	:	$\times 30$ 90	$\times 30$ 150												

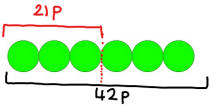
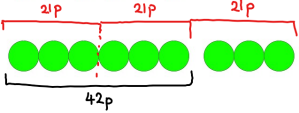
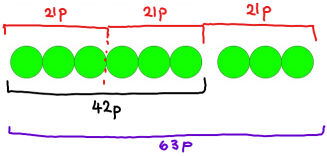
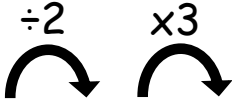
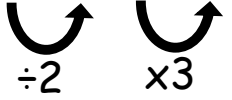
COMPOUND MEASURES

GENERAL TEACHING & LEARNING POINTS	<ul style="list-style-type: none"> Students should have a secure understanding of units and what these units mean. E.g. mph is "how many miles something travels per hour" or Pressure "How many Newtons per every 1 cm²" Students should apply their understanding of proportion when working with Compound Measures Try to avoid using formula triangles with Compound Measures as these do not promote teaching for understanding. 																						
Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract																				
N/A	Students should use a bar model to represent the problem.	Students should use a proportion table to scale up or down each quantity.	Students should confidently state and use the Compound Measures formulae without the need for an alternative representation.																				
Example(s)	Example(s) - These should be seen in books																						
A car travels at a speed of 45 mph for 20 minutes. How far does it travel in this time?																							
NOT APPLICABLE	<p>Students should represent this as a bar model.</p> <p>Notice that the distance travelled in 1 hour is represented using equal length bars.</p> <table border="1" style="margin: 5px auto; border-collapse: collapse;"> <tr><td style="text-align: center;">45 miles</td></tr> <tr><td style="text-align: center;">1 hour</td></tr> </table> <p>Students should then understand that 20 minutes is one third of an hour and therefore this needs to be split into three equal parts.</p> <p>Students should then use proportion to calculate the distance travelled in this time.</p> <table border="1" style="margin: 5px auto; border-collapse: collapse;"> <tr><td style="text-align: center;">15 miles</td><td style="text-align: center;">15 miles</td><td style="text-align: center;">15 miles</td></tr> <tr><td colspan="3" style="text-align: center;">45 miles</td></tr> <tr><td colspan="3" style="text-align: center;">1 hour</td></tr> <tr><td style="text-align: center;">20 mins</td><td style="text-align: center;">20 mins</td><td style="text-align: center;">20 mins</td></tr> </table> <p>Students should then be able to identify that in 20 minutes the car travelled 15 miles.</p>	45 miles	1 hour	15 miles	15 miles	15 miles	45 miles			1 hour			20 mins	20 mins	20 mins	<p>At this stage students should use a proportion table to determine the unknown quantity.</p> <div style="text-align: center; margin: 10px 0;"> $\div 3$  </div> <table border="1" style="margin: 5px auto; border-collapse: collapse;"> <tr><td style="text-align: center;">Distance</td><td style="text-align: center;">45 miles</td><td style="text-align: center;">15 miles</td></tr> <tr><td style="text-align: center;">Time</td><td style="text-align: center;">1 hour</td><td style="text-align: center;">$\frac{1}{3}$ hour</td></tr> </table> <div style="text-align: center; margin: 10px 0;"> $\div 3$  </div>	Distance	45 miles	15 miles	Time	1 hour	$\frac{1}{3}$ hour	<p>Students should use and substitute into the formula.</p> <p>Avoid using the formula triangles, however, this should be the only stage at which then could be used if students are unable to understand the basic concepts after significant time has been spent.</p> <p>Students should clearly state the formula they are using (in its original form) followed by any re-arrangement as a separate stage of working.</p> <p>Students should state the value of quantities in the question, with particular emphasis on units.</p> <p style="text-align: center;">$S = 45 \text{ mph} \quad t = \frac{1}{3} \text{ hour}$</p> <p style="text-align: center;">$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$</p> <p style="text-align: center;">Distance = Speed x Time</p> <p style="text-align: center;">Distance = $45 \times \frac{1}{3} = \mathbf{15 \text{ miles}}$</p>
45 miles																							
1 hour																							
15 miles	15 miles	15 miles																					
45 miles																							
1 hour																							
20 mins	20 mins	20 mins																					
Distance	45 miles	15 miles																					
Time	1 hour	$\frac{1}{3}$ hour																					

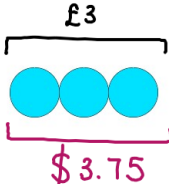
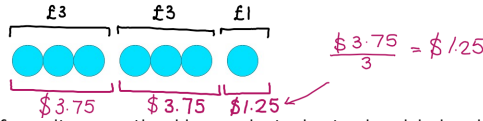
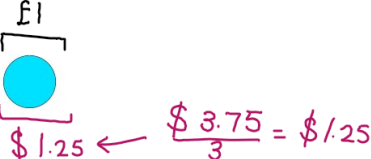
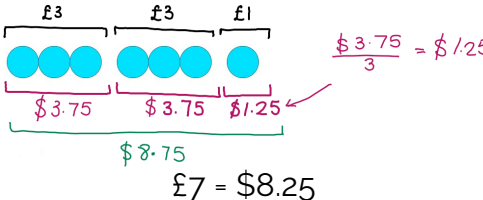
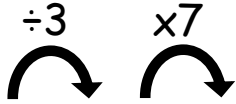
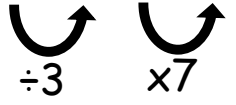
DIRECT PROPORTION

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract																																																																															
Students should be able to represent this problem using counters or multi-link cubes.	Students should be able to represent the information using a bar model in their books.	Students should use a direct proportion table to be able to represent and solve the problem	Students should be able to use direct proportion using the same format as simplifying a ratio.																																																																															
Example(s)	Example(s) - These should be seen in books																																																																																	
7 pens cost 42p. What is the cost of 1 pen?																																																																																		
<p>Students should represent the problem by using 7 counters or cubes to represent the 7 pens.</p>  <p style="text-align: center;">42p</p> <p>Students should then be able to work out the value of 1 pen.</p> <p>Higher attaining students will be able to spot that, for other problems, that it isn't always necessary to find the value of 1 part first. E.g. 8 pens cost 80p work out the cost of 2 pens. Students will be able to divide by 4 in one step rather than dividing by 8 then multiplying by 2.</p>  <p style="text-align: center;">42p</p> <p>● = $\frac{42p}{7} = 6p$</p> <p>Students should then clearly answer the question.</p> <p style="text-align: center; font-size: 1.2em;">1 pen = 6p</p>	<p>Students should draw the diagram below in their books.</p> <table border="1" style="width: 100%; text-align: center; border-collapse: collapse;"> <tr> <td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td> </tr> <tr> <td colspan="8">42p</td> </tr> </table> <p>Students should then clearly show that they have divided the 42p into 7 parts by adding this to their diagram and showing the division calculation.</p> <table border="1" style="width: 100%; text-align: center; border-collapse: collapse;"> <tr> <td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td> </tr> <tr> <td colspan="8">42p</td> </tr> <tr> <td>6p</td><td>6p</td><td>6p</td><td>6p</td><td>6p</td><td>6p</td><td>6p</td><td>6p</td> </tr> </table> <p style="text-align: center;">$\frac{42p}{7} = 6p$</p> <p>Students could then circle the part of their diagram that gives the value of 1 pen.</p> <table border="1" style="width: 100%; text-align: center; border-collapse: collapse;"> <tr> <td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td> </tr> <tr> <td colspan="8">42p</td> </tr> <tr> <td>6p</td><td>6p</td><td>6p</td><td>6p</td><td>6p</td><td>6p</td><td>6p</td><td>6p</td> </tr> </table> <p style="text-align: center;">$\frac{42p}{7} = 6p$</p> <p>Students should then clearly answer the question.</p> <p style="text-align: center; font-size: 1.2em;">1 pen = 6p</p>	1 Pen	1 Pen	1 Pen	1 Pen	1 Pen	1 Pen	1 Pen	1 Pen	42p								1 Pen	1 Pen	1 Pen	1 Pen	1 Pen	1 Pen	1 Pen	1 Pen	42p								6p	6p	6p	6p	6p	6p	6p	6p	1 Pen	1 Pen	1 Pen	1 Pen	1 Pen	1 Pen	1 Pen	1 Pen	42p								6p	6p	6p	6p	6p	6p	6p	6p	<p>At this stage students should use a proportion table to determine the unknown quantity.</p> <div style="text-align: center;"> $\div 7$  </div> <table border="1" style="width: 100%; text-align: center; border-collapse: collapse;"> <tr> <td style="width: 25%;">Pens</td> <td style="width: 25%;">7 pens</td> <td style="width: 25%;">1 pen</td> </tr> <tr> <td>Cost</td> <td>42p</td> <td>6p</td> </tr> </table> <div style="text-align: center;">  </div> <p>Students should then clearly answer the question.</p> <p style="text-align: center; font-size: 1.5em;">1 pen = 6p</p>	Pens	7 pens	1 pen	Cost	42p	6p	<p>Students should represent the problem in a table.</p> <p>Students should clearly show that the division operations need to be applied to both sides to represent the direct proportionality.</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td>Pens</td> <td>:</td> <td>Cost</td> </tr> <tr> <td>7 pens</td> <td>:</td> <td>42p</td> </tr> <tr> <td>$\div 7$ 1 pen</td> <td>:</td> <td>6p $\div 7$</td> </tr> </table> <p>Students should then clearly answer the question.</p> <p style="text-align: center; font-size: 1.5em;">1 pen = 6p</p>	Pens	:	Cost	7 pens	:	42p	$\div 7$ 1 pen	:	6p $\div 7$
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BEST BUY PROBLEMS

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract																																																																																																
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<p>Shop A sells 6 pens for 42p. Shop B sells 9 pens for 72p. Bella wants to buy some pens from one of the shops. Which shop should Bella buy the pens from?</p>																																																																																																			
<p>Students should represent the problem by using 7 counters or cubes to represent the 7 pens.</p> <p>Students should then be able to work out the value of 3 pens.</p>  <p>Students could use a unitary approach here, however, students should be encouraged to be efficient with their method and therefore since 9 is a multiple of 3 this would be more effective.</p> <p>Students should then add in the additional counters to their diagram in order to work out the cost of an equivalent number of pens as Shop B but from Shop A.</p>  <p>Students should then work out the cost from Shop A</p>  <p>Students should then clearly answer the question.</p> <p style="text-align: center; font-size: 1.2em;">Bella should buy from Shop A</p>	<p>Students should draw similar diagrams to the Physical Stage below in their books, this time using a bar model</p> <table border="1" style="margin-bottom: 10px; width: 100%; text-align: center;"> <tr><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td></tr> <tr><td colspan="6">42p</td></tr> </table> <table border="1" style="margin-bottom: 10px; width: 100%; text-align: center;"> <tr><td colspan="3">21p</td><td colspan="3">21p</td></tr> <tr><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td></tr> <tr><td colspan="6">42p</td></tr> </table> <table border="1" style="margin-bottom: 10px; width: 100%; text-align: center;"> <tr><td colspan="2">21p</td><td colspan="2">21p</td><td colspan="2">21p</td></tr> <tr><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td></tr> <tr><td colspan="4">42p</td><td colspan="2">21p</td></tr> </table> <table border="1" style="margin-bottom: 10px; width: 100%; text-align: center;"> <tr><td colspan="2">21p</td><td colspan="2">21p</td><td colspan="2">21p</td></tr> <tr><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td></tr> <tr><td colspan="4">42p</td><td colspan="2">21p</td></tr> <tr><td colspan="6">63p</td></tr> </table> <p>Students should then clearly answer the question.</p> <p style="text-align: center; font-size: 1.2em;">Bella should buy from Shop A</p>	1 Pen	1 Pen	1 Pen	1 Pen	1 Pen	1 Pen	42p						21p			21p			1 Pen	1 Pen	1 Pen	1 Pen	1 Pen	1 Pen	42p						21p		21p		21p		1 Pen	1 Pen	1 Pen	1 Pen	1 Pen	1 Pen	42p				21p		21p		21p		21p		1 Pen	1 Pen	1 Pen	1 Pen	1 Pen	1 Pen	42p				21p		63p						<p>At this stage students should use a proportion table to determine the cost of pens in Shop A.</p> <div style="text-align: center; margin-bottom: 10px;"> $\div 2$ $\times 3$  </div> <table border="1" style="margin-bottom: 10px; width: 100%; text-align: center;"> <tr><th colspan="4">Shop A</th></tr> <tr><td>Pens</td><td>6 pens</td><td>3 pens</td><td>9 pens</td></tr> <tr><td>Cost</td><td>42p</td><td>21p</td><td>63p</td></tr> </table> <div style="text-align: center; margin-bottom: 10px;"> $\div 2$ $\times 3$  </div> <p style="text-align: center; font-size: 1.2em;">Bella should buy from Shop A</p>	Shop A				Pens	6 pens	3 pens	9 pens	Cost	42p	21p	63p	<p>Students should represent the problem in a table.</p> <p>Students should clearly show that the division operations need to be applied to both sides to represent the direct proportionality.</p> <div style="text-align: center; margin-bottom: 10px;"> <u>Shop A</u> </div> <table style="margin-left: auto; margin-right: auto;"> <tr><td>Pens</td><td>:</td><td>Cost</td></tr> <tr><td>6 pens</td><td>:</td><td>42p</td></tr> <tr><td>$\div 2$ 3 pens</td><td>:</td><td>21p</td></tr> <tr><td>$\times 3$ 9 pens</td><td>:</td><td>63p</td></tr> </table> <p style="text-align: center; font-size: 1.2em;">Bella should buy from Shop A</p> <p>Students should then clearly answer the question.</p>	Pens	:	Cost	6 pens	:	42p	$\div 2$ 3 pens	:	21p	$\times 3$ 9 pens	:	63p
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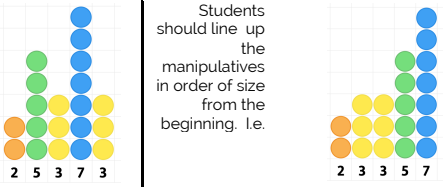
EXCHANGE RATES

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract																																					
Students should be able to represent this problem using counters or multi-link cubes.	Students should be able to represent the information using a bar model in their books.	Students should use a direct proportion table to be able to represent and solve the problem	Students should be able to use direct proportion using the same format as simplifying a ratio.																																					
Example(s)	Example(s) - These should be seen in books																																							
<p>The exchange rate from Pounds (£) into US Dollars (\$) is £3 = \$3.75. Calculate how many Dollars (\$) are equivalent to £7.</p>																																								
<p>Students should be able to represent the initial problem and information with a manipulative.</p> <div style="text-align: center;">  </div> <p>Students should then be able to scale up the given quantity, by either using a unitary method or by understanding that £7 is 2 x £3 + £1, as shown:</p> <div style="text-align: center;">  </div> <p>If a unitary method is used, students should clearly show how they obtained the value of a single unit.</p> <div style="text-align: center;">  </div> <p>Students should then be able to add up the corresponding amounts and give a final answer.</p> <div style="text-align: center;">  </div>	<p>Students should draw similar diagrams to the Physical Stage below in their books, this time using a bar model.</p> <table border="1" style="margin: 10px auto; border-collapse: collapse;"> <tr><td style="text-align: center;">£3</td></tr> <tr><td style="text-align: center;">\$3.75</td></tr> </table> <p>Students should clearly show how they are scaling up the value.</p> <table border="1" style="margin: 10px auto; border-collapse: collapse;"> <tr> <td style="text-align: center;">£3</td> <td style="text-align: center;">£3</td> <td style="text-align: center;">£1</td> </tr> <tr> <td style="text-align: center;">\$3.75</td> <td style="text-align: center;">\$3.75</td> <td style="text-align: center;">\$1.25</td> </tr> </table> $£1 = \frac{\$3.75}{3} = \1.25 <p>Students should then show the total value and clearly state their final answer.</p> <table border="1" style="margin: 10px auto; border-collapse: collapse;"> <tr> <td style="text-align: center;">£3</td> <td style="text-align: center;">£3</td> <td style="text-align: center;">£1</td> </tr> <tr> <td style="text-align: center;">\$3.75</td> <td style="text-align: center;">\$3.75</td> <td style="text-align: center;">\$1.25</td> </tr> <tr> <td colspan="3" style="text-align: center;">\$8.25</td> </tr> </table> <p style="text-align: center; font-size: 1.2em;">£7 = \$8.25</p>	£3	\$3.75	£3	£3	£1	\$3.75	\$3.75	\$1.25	£3	£3	£1	\$3.75	\$3.75	\$1.25	\$8.25			<p>At this stage students should use a proportion table to determine the cost of pens in Shop A.</p> <div style="text-align: center; margin-bottom: 10px;">  </div> <table border="1" style="margin: 10px auto; border-collapse: collapse;"> <caption style="text-align: center;">Shop A</caption> <tr> <td style="text-align: center;">£</td> <td style="text-align: center;">£3</td> <td style="text-align: center;">£1</td> <td style="text-align: center;">£7</td> </tr> <tr> <td style="text-align: center;">\$</td> <td style="text-align: center;">\$3.75</td> <td style="text-align: center;">\$1.25</td> <td style="text-align: center;">\$8.25</td> </tr> </table> <div style="text-align: center; margin-top: 10px;">  </div> <p>Students should then clearly state their final answer.</p> <p style="text-align: center; font-size: 1.2em;">£7 = \$8.25</p>	£	£3	£1	£7	\$	\$3.75	\$1.25	\$8.25	<p>Students should represent the problem in a table.</p> <p>Students should clearly show that the division operations need to be applied to both sides to represent the direct proportionality.</p> <table style="margin: 10px auto; border-collapse: collapse;"> <tr> <td style="text-align: center;">£</td> <td style="text-align: center;">:</td> <td style="text-align: center;">\$</td> </tr> <tr> <td style="text-align: center;">£3</td> <td style="text-align: center;">:</td> <td style="text-align: center;">\$3.75</td> </tr> <tr> <td style="text-align: center;">÷3</td> <td style="text-align: center;">:</td> <td style="text-align: center;">\$1.25</td> </tr> <tr> <td style="text-align: center;">x7</td> <td style="text-align: center;">:</td> <td style="text-align: center;">\$8.25</td> </tr> </table> <p>Students should then clearly state their final answer.</p> <p style="text-align: center; font-size: 1.2em;">£7 = \$8.25</p>	£	:	\$	£3	:	\$3.75	÷3	:	\$1.25	x7	:	\$8.25
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MEAN, MODE, MEDIAN AND RANGE

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
Students should be able to represent a problem using manipulatives (counters or multi-link cubes). It may be useful for students to use different colours for the different values.	Students should draw the manipulatives as a diagram in their books.	Students should be comfortable with the formal definitions of mode, mean, median and range at this point.	Students should be able to find the mean, mode, median and range without needing a diagram. Higher attaining students should link to algebra during working.
Example(s)	Example(s) - These should be seen in books		

Find the mode, range, median and mean of 2, 5, 3, 7, 3.




Students should line up the manipulatives in order of size from the beginning. I.e.

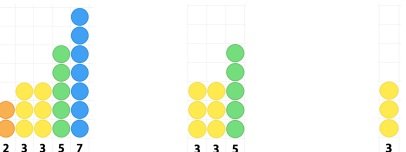
Mode
If different colours have been used this is the colour used for the most columns. If colours aren't used then students should use which height is the most common.

Range
Students should count the difference between the biggest value and the smallest value.

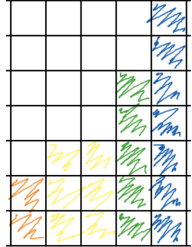
Mean
Students should aim to level out the manipulatives into a rectangle, removing one from the largest to the smallest. The mean is the height of each bar at the end of the process. It is important to highlight that the mean can be a decimal at this point.



Median
Students should remove the smallest and biggest column and repeat. The median is the column that is remaining. Where there are two columns remaining students should find the mean of the two columns as above.

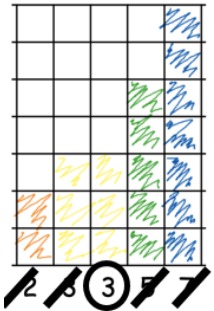


Students should represent the numbers as bars using the squares in their books. This should be smallest to largest.



Mode, Range and Mean
As Physical Stage.

Median
Students should cross off the numbers below the bars smallest then largest. Students should circle the median value.



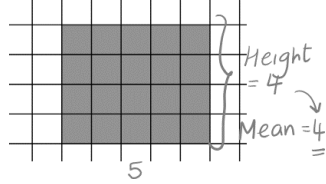
Mode
Students at this point should know the mode is the most common value and should be able to determine this by inspection from the list of numbers.

Range
Students at this point should know the range is **Largest - Smallest**
Students should clearly show this calculation, i.e.

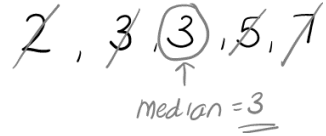
$$7 - 2 = 5$$

Mean
At this point students should have identified the relationship that the total number of blocks/counters forms the area of the rectangle and the mean is the height of the rectangle.

This should lead to the understanding that the mean is the total number divided by the amount of numbers.



Median
Students at this point should know the median in the middle number.
Students should be able to find the median without the aid of a diagram.
Students should cross off the numbers, smallest then largest. Students should circle the median value.



Mode
As semi-abstract stage.

Range
As semi-abstract stage.

Mean
Students should be confident with the definition and calculating the mean as below:

$$\frac{2 + 3 + 3 + 5 + 7}{5}$$

Higher attaining students should be introduced to the formula written using sigma and notation, i.e.

Median
Students at this point should know the median in the middle number.
Students should be able to state which number is the middle number using the formula

$$\text{Median Number} = \frac{n + 1}{2}$$

In this case:

$$\frac{n + 1}{2} = \frac{5 + 1}{2} = 3$$

So the median is the 3rd number in the list when the values are put in order.